2 Rapid eustatic changes—never globally uniform

Willy Fjeldskaar

Rogaland Research Institute, PO Box 2503, N 4004 Stavanger, Norway

The classical definition of eustasy is vertical movements of sea level, which were originally believed to be worldwide simultaneous uniform changes. However, changes in distribution of the Earth's mass bring about geoid changes, and resulting changes of sea level may vary significantly over the globe. Accordingly, it is difficult to imagine any significant sea-level changes which are globally uniform.

The geoid is defined as the equipotential surface of the Earth's gravitational field that corresponds to the sea level. Any process causing sea-level movements changes the gravity field. This chapter focuses on the supposedly most important cause of rapid sea-level variations, namely glaciation/deglaciation.

Calculations of gravity changes due to glaciation/deglaciation show that the resulting sea-level movements differ significantly over the globe. During a deglaciation there is no uniform worldwide rise in sea level due to an increased volume of ocean water; in the region near the former glacier there is actually a fall in sea level. During the deglaciation in Scandinavia, the sea level perhaps fell several tens of meters. The area where the sea level falls has a lateral extent of several hundred kilometers. Outside this area the sea level rises.

Furthermore, it is demonstrated that rapid sea-level changes, regardless of cause, give rise to processes which alter the geoid. Rapid eustatic changes are thus never uniform over the globe. Processes of more long-term eustatic change, such as changes in the ocean ridge systems and small-scale sedimentation, probably cannot take place either without altering the geoid.

Geoidal eustasy is an important effect, and must be taken into account in correlations of sea level changes. This implies that any eustatic curve claiming to be global may be questioned.

INTRODUCTION

The word 'eustasy' (Greek, *ev* = good, *stasis* = position) was first proposed by Eduard Suess (1888). His studies of Tertiary stratigraphy indicated that transgressions took place simultaneously all over Europe. Marine sediments above the present sea level were explained by movements of the ocean level. However, while the sea-level record was influenced, to a certain extent, by local tectonics, Suess found no sign of large-scale vertical movements of the solid Earth. He assumed that changes in sea level were caused by climatic variations, tectonic movements, volcanism and, most importantly, sedimentation.

Eustasy is commonly defined as globally uniform sea-level changes. The theory of classical eustasy (Fairbridge, 1961) distinguished between different types of eustatic changes of sea level:

(a) tectono-eustasy caused by volcanism, development of fold belts and oceanic trenches and isostatic movements;
(b) sedimento-eustasy caused by sedimentation;
(c) glacio-eustasy caused by glaciations;
(d) other eustatic processes, such as thermal expansion/contraction of the ocean water and production of water by volcanoes.

Great effort has been spent on constructing global eustatic curves for the Quaternary (e.g. Fairbridge, 1961) as well as for the entire Phanerozoic. The most well known curves for long-term changes are the 'Exxon curves' (Vail *et al.*, 1977; Haq *et al.*, 1987). The methodology upon which these curves are based (stratigraphic models and chronostratigraphic interpretations), has been criticized and the global universality of the curves has been questioned (e.g. Miall, 1986). The intention of this chapter is to show, theoretically and numerically, that the processes causing rapid eustatic changes do not operate without significantly influencing the gravity field, resulting in non-uniform sea-level changes over the globe. We will focus on the most important causes of eustatic change, glaciations, sedimentation and variations of the ocean ridge system. The non-uniformity of the rapid eustatic change is illustrated by a series of calculations of the geoid changes associated with the last glaciation in Europe.

It is also shown that rapid changes in sea level, such as

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those reported by Vail *et al.* (1977), whatever their cause, lead to subsidence of the ocean floor and to significant uplift of the continents. The crustal movements affect the geoid, giving non-uniform eustatic changes.

THE GEOID

It was already well known in the 19th century that the mass of mountains causes considerable deflections of the plumb line. Pratt (1855) claimed that this would cause a rise in sea level in the vicinity of the mountains, and presented numerical calculations of this effect. The irregular ocean level was later named 'the geoid' (Listing, 1873).

The geoid is an equipotential surface of the Earth's gravitational field, corresponding to sea level. Sea level can be measured only over oceans, but the geoid is a complete, closed surface. Under the continents the geoid can be thought of as the surface defined by the water level in narrow canals cut to sea level through the land masses.

Where there are local variations in *g*, due to internal density anomalies, the geoid is distorted. A mathematical figure representing the sea-level surface with all irregularities removed is named the spheroid. This would be the sea-level surface of an Earth with no lateral variations in density. The difference in elevation between the measured geoid and the spheroid is called the geoid anomaly. A map of the geoid anomaly is shown in Fig. 1.

The geoid configuration is not stable, but changes over time due to changes in the Earth's gravitational field. Some of the major anomalies illustrated in Fig. 1, such as the geoid high over New Guinea or the geoid low over India, are probably related to mantle convection or plate-tectonic phenomena (cf. Chase, 1979; Ricard *et al.*, 1988).

EUSTATIC CHANGE

Bearing in mind the geoid changes caused by changes in the Earth's mass distribution, the concept of eustasy is not straightforward. Sea level is, in general, not parallel with the Earth's spheroid, and is not stable. Geoid changes are brought about both by external (climate changes, sedimentation, etc.) and internal (mantle flow, core/mantle changes etc.) processes.

An up-to-date definition of eustasy is vertical changes of sea level, regardless of causation (Mörner, 1976), and these are certainly not globally uniform (except for sea-level effects due to possible changes in the volume of the hydrosphere or expansion/contraction of the Earth). Eustasy is of three types:

(1) Glacial-eustasy is controlled by variation of the ocean water volume. The most effective mechanism for such variations is, of course, glaciation/deglaciation. These variations are of short timescale, resulting in rapid sea-level fluctuations, and would account for the oscillations of sea level inferred for the Oligocene–Miocene and onwards (Tanner, 1968; Rona, 1973), for the Permian and for the Ordovician (Mörner, 1987). In addition, oxygen-isotope data indicate ice build-up even in the early Tertiary (Matthews, 1983). Glacial-eustatic changes thus provide an explanation for most of the third-order cycles of the 'Exxon curves'.

(2) Tectono-eustasy is controlled by variation of the ocean basin volume. The most important factors in such changes are sedimentation (Hays and Pitman, 1973) and variations in the volume of ocean ridge systems (Pitman, 1978). Intraplate stresses (Cloetingh *et al.*, 1985) may provide a tectonic mechanism for short-term sea-level changes.

(3) Geoidal eustasy is caused by variations in the Earth's gravitational field.

DEGLACIATION AND THE GEOID

Geoidal eustasy is change in the ocean water distribution. It affects the ocean level globally, but the sign and magnitude of changes differ over the globe. The importance of the geoidal eustatic process will now be illustrated by a theoretical simulation of geoid deformation connected to deglaciation.

Change in the shape of the geoid is an often overlooked effect of deglaciation, although several papers have described this effect. The first author to do so was Penck (1882). He was followed by Woodward (1888), who made numerical calculations of the effect and concluded that it

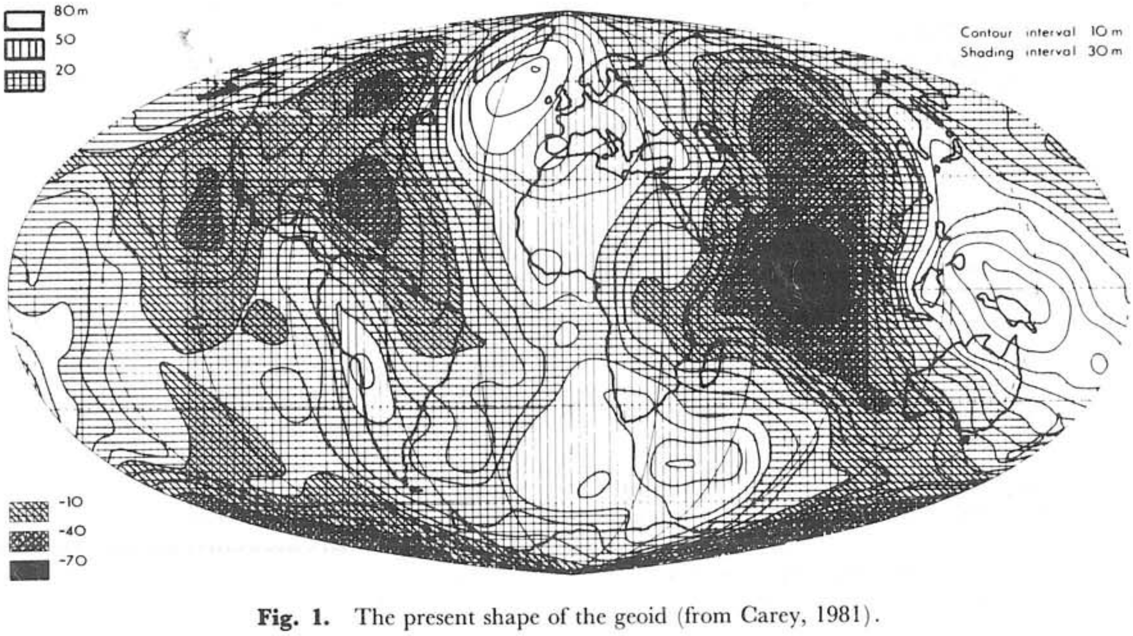

Fig. 1. The present shape of the geoid (from Carey, 1981).

was certainly significant. The effect then seems to have been forgotten for many years, until Jensen (1972) revived the subject. In recent years several authors writing about postglacial uplift and sea-level changes have taken this effect into account (e.g. Cathles, 1975, 1980; Clark, Farrell and Clark, 1976; Clark, Farrell and Peltier, 1978; Fjeldskaar, 1978, 1981; Fjeldskaar and Kanestrøm, 1980; Peltier, 1980; Wolf, 1985; Nakada and Lambeck, 1987). As most of these studies are concerned with global postglacial sea-level changes, the importance of geoidal eustasy near previous ice sheets is not easily extracted.

In the following calculations, the isolated effect of geoidal eustasy is illustrated in connection with glaciation/deglaciation and related crustal uplift in Fennoscandia and the Barents Sea. The glaciation data used in the calculations are based on the last glaciation, but the resulting geoidal effect can be applied to other glaciation periods as well. In fact, the extent of the last great ice sheet in Europe seems to be small than previous ice sheets, and is certainly smaller than the Canadian ice sheet.

The maximum extent (at 20 000 years BP) of the last ice sheet in Europe is shown in Fig. 2 (Denton and Hughes, 1981). Let us first assume that the ice sheet melts rapidly. This is not what happened in this particular case (rapid deglaciation might, however, have occurred in other periods), but it gives us a feeling for the geoidal effect. The calculations are carried out using a Fourier transform technique, using equation (3) of the Appendix. The calculated geoidal eustatic effect in central areas of the ice sheet is close to 100 m (Fig. 3). This is the fall of the geoid caused by a rapid (instantaneous) deglaciation, or the rise of the geoid caused by a rapid glaciation. Notice also the large difference in geoidal effect in the North Sea and the Barents Sea, up to 30–40 m.

However, a realistic simulation of the geoidal eustatic effects connected to the deglaciation requires consideration of the deglaciation history. From the marginal moraines of different ages we know that deglaciation took place over a time span of about 10 000 years. The deglaciation history used (Figs 2 and 4–6) was compiled by B. G. Andersen (Denton and Hughes, 1981). During deglaciation the

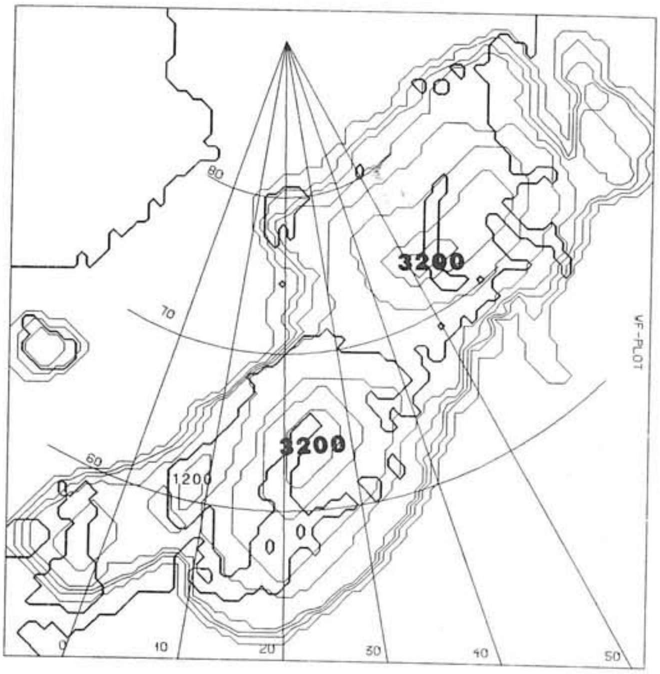

Fig. 2. The extent and thickness of the ice sheet in Europe at the last glacial maximum (20 000 BP). The contour interval is 400 m (redrawn after Denton and Hughes, 1981).

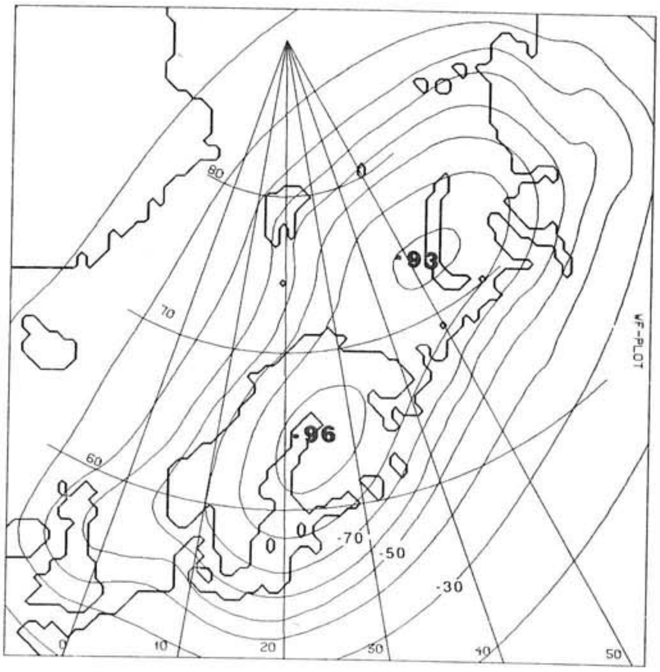

Fig. 3. Theoretical deflection of the geoid caused by an instantaneous deglaciation of the ice sheet of Fig. 2. The contour interval is 10 m.

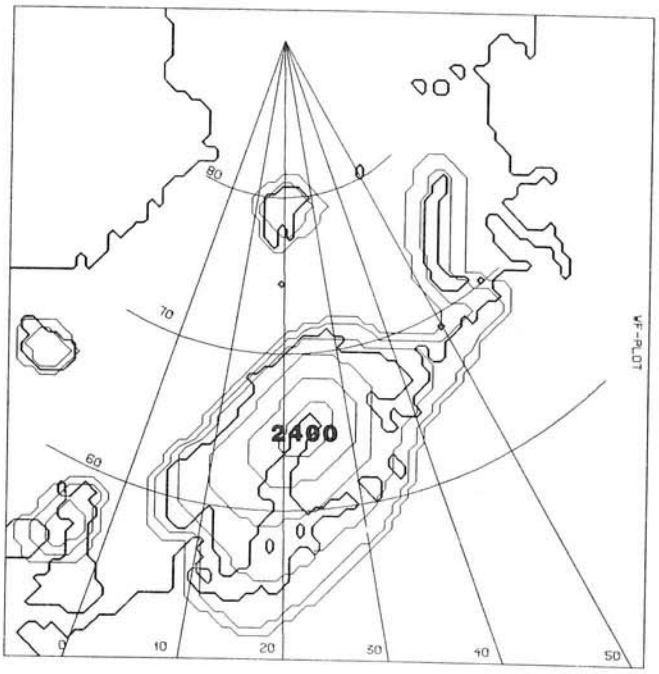

Fig. 4. The ice sheet at 15 000 BP. The contour interval is 400 m (ice sheet margins are redrawn after Denton and Hughes, 1981).

crust is uplifted to approach a new state of equilibrium consistent with the removal of ice: this is the isostatic process. When there is a very slow deglaciation, the related geoidal eustatic deflections tend to be smeared out, because lost ice mass is compensated for by increased crustal masses.

Cathles (1975) laid the theoretical foundations necessary to model the isostatic adjustments of a (Maxwell) visco-elastic Earth, in which viscosity varies with depth. A flat-Earth approximation is used here. A study of postglacial uplift in Scandinavia has revealed the necessary information about the lithosphere and mantle to permit a simulation of the geoidal eustasy in the area (Fjeldskaar and Cathles,

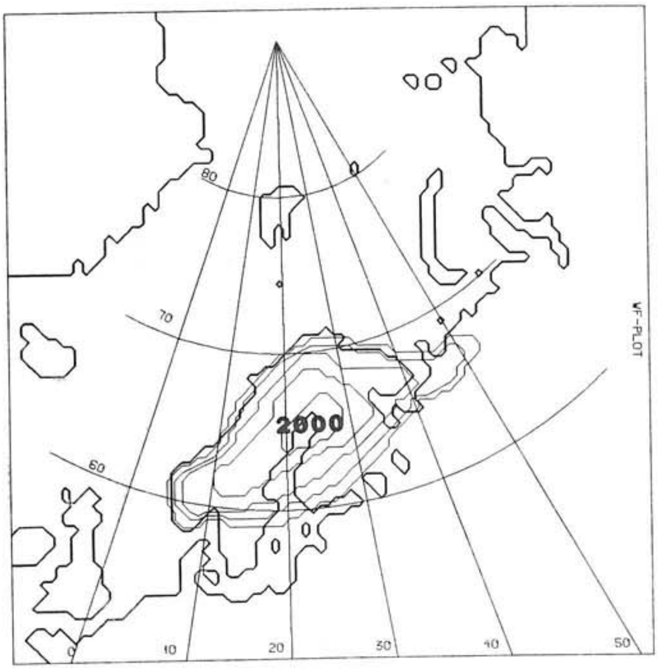

Fig. 5. The ice sheet at 10 000 BP. The contour interval is 400 m (ice sheet margins are redrawn after Denton and Hughes, 1981).

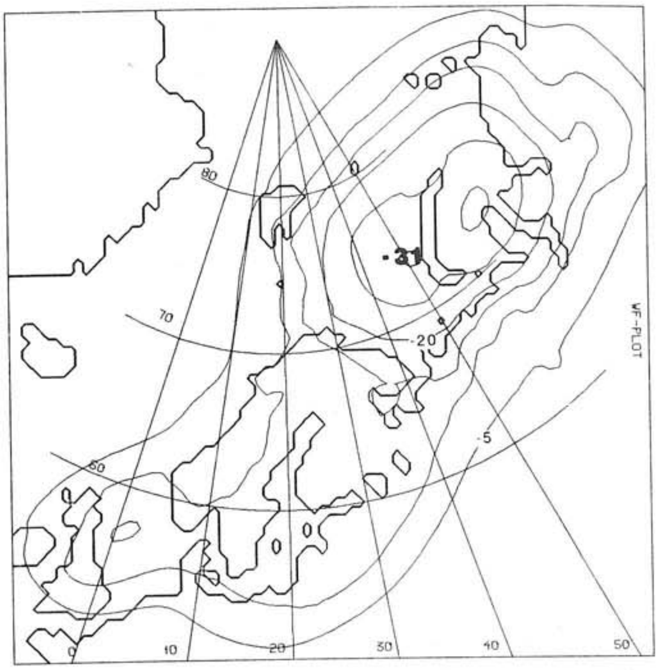

Fig. 7. Theoretical geoid deformation at 15 000 BP caused by the observed deglaciation. The contour interval is 5 m

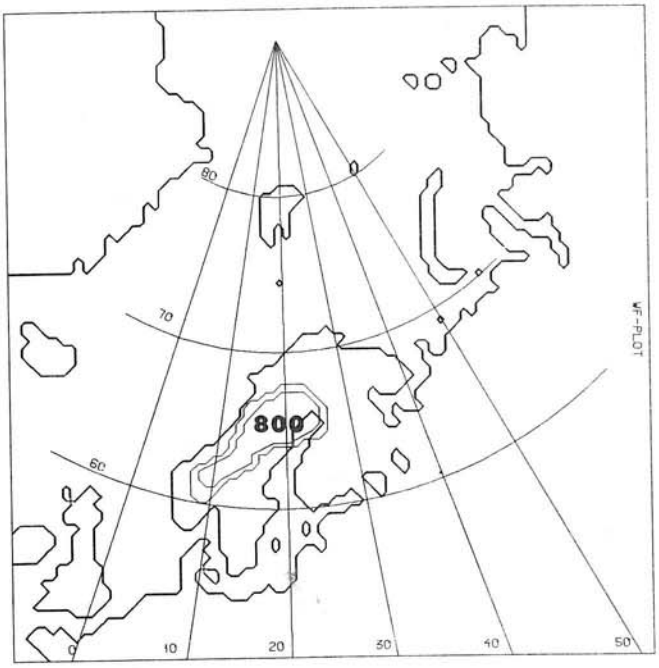

Fig. 6. The ice sheet at 9300 BP. The contour interval is 400 m (ice sheet margins are redrawn after Denton and Hughes, 1981).

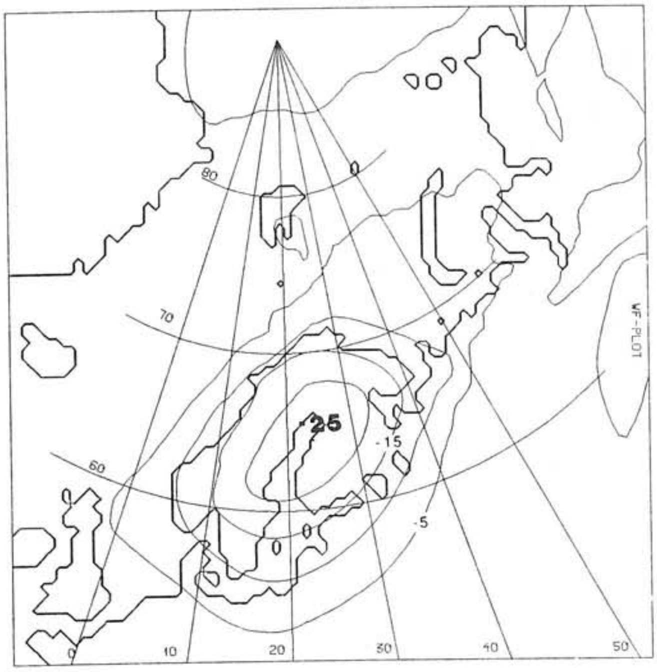

Fig. 8. Theoretical geoid deformation at 9000 BP caused by the observed deglaciation. The contour interval is 5 m.

1987). In that study it was shown that the best fit with the observed present rate of uplift in Norway is achieved with a lithosphere of mechanical thickness 80 km and a mantle viscosity of 1.0×10^{22} poise, except for a 75 km asthenosphere of viscosity 1.3×10^{20} poise.

The ice sheet is assumed to have been in isostatic equilibrium prior to 20 000 BP. Thus there is no geoid deflection in the area. Melting of the ice started at 20 000 BP and continued until the area became ice-free at 8500 BP. The melting from one ice sheet configuration to the next (shown in Figs 2 and 4–6) is modelled with uniform speed. The theoretical history of geoidal eustatic change based on this Earth rheology and the above glacial history is presented in Figs 7 and 8.

The modelled ice melting started at 20 000 BP, mainly in the Barents Sea, and at 19 000 BP there is a growing geoid deformation in this area. The deformation in the Barents Sea increases to about 31 m at 15 000 BP (Fig. 7). After 15 000 BP the deformation begins to decrease, while deformation in Scandinavia is developing. At 9000 BP the geoid is almost back to normal in the Barents Sea, and at the same time the geoid deformation in Scandinavia is at a maximum of about 25 m (Fig. 8). From 9000 BP to the present the geoid slowly returns to the undeformed state. At present the geoid is approximately 2 m from equilibrium.

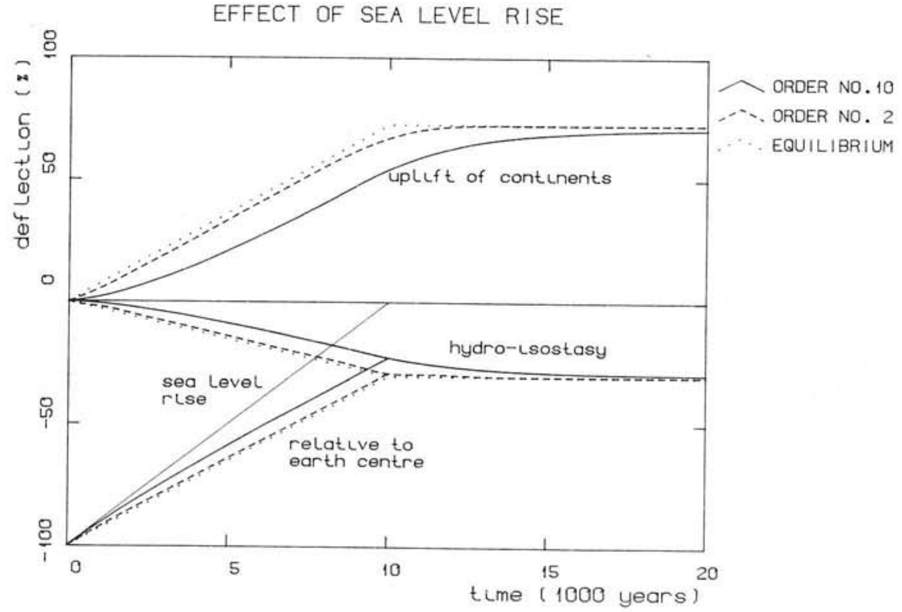

Fig. 9. Effects of sea-level rise, regardless of causation. The sea-level rise is assumed to take place at uniform speed over 10 000 years. The horizontal axis shows time after the start of sea-level change, and the vertical axis shows deflections in the percentage of total sea-level rise.

GLACIAL EUSTASY IN GENERAL

Let us now look at glacial eustasy from a more general global point of view. Any sea-level change causes deflection of the ocean floor, hydro-isostasy, to attain isostatic equilibrium. The hydro-isostasy is approximately one-third of the sea-level change. The continents are simultaneously deflected, with a mean magnitude over the continents that is twice the deflection of the ocean floor. This is due to the fact that the oceanic area is double the land area. These points are illustrated in Fig. 9.

The process of hydro-isostasy and its influence on sea level has been examined by several authors (e.g. Walcott, 1972; Chappell, 1974). An interesting implication of hydro-isostasy is the fact that the sea-level history will differ between oceanic islands and continental margins. An island moving with the sea floor will record the full sea-level change, while points near the continents record quite different sea-level changes. Thus hydro-isostasy is an important factor in determining relative sea-level fluctuations.

Another interesting consequence of hydro-isostasy is the fact that neither the ocean floor nor the land masses will achieve isostatic equilibrium during rapid sea-level changes, regardless of causation. The degree of compensation varies with the wavelength of the load. The mean deflection of the ocean floor and the continents, calculated for order no. 2 (wavelength 16 000 km) and order no. 10 (wavelength 3800 km), is illustrated in Fig. 9. For order no. 2 (relaxation time 800 years; Fjeldskaar and Cathles, unpublished result) there is close isostatic equilibrium during the sea-level change. For order no. 10 (relaxation time 3000 years; Fjeldskaar and Cathles, unpublished result) the deflection is clearly delayed compared to the sea-level change, giving rise to geoidal deflections over the oceans as well as over the continents.

TECTONO-EUSTASY AND THE GEOID

As mentioned above, the most important factors influencing changes of the ocean basin volume are sedimentation and variations in the volume of ocean ridge systems.

Sedimentation

Sedimentation is a much slower process than glaciation/deglaciation, giving the crust a chance to maintain isostatic equilibrium during deposition. However, this is the case only when the deposits are of long wavelength. A load of short wavelength will be balanced entirely by the lithosphere. The wavelength of the surface load is so small that the lithosphere is effectively infinite, and no buoyant forces are active. For a lithosphere of flexural rigidity 5×10^{23} Nm and a load wavelength of less than 200 km, the load is more than 95% supported by the lithosphere (Fig. 10) (Fjeldskaar and Pallesen, 1989). Then there is only negligible isostatic movement, and any mass anomalies cause deflections of the geoid, according to the equations (1)–(3) (see the Appendix).

Assuming deposition within a small circular sedimentary basin, with a radius of 50 km, a thickness of 3 km and a sediment density of $2\,g\,cm^{-3}$, a geoid deflection of 13 m above the centre of the basin, decreasing radially, will form, according to equation (2).

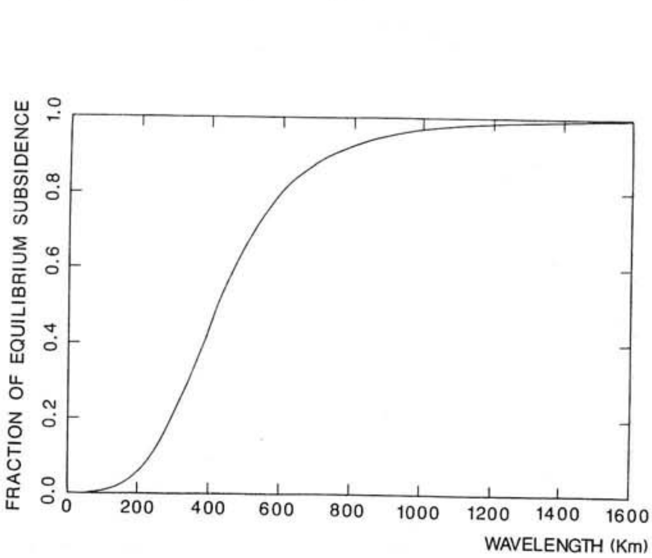

Fig. 10. Subsidence of the base of the lithosphere, relative to equilibrium subsidence. The flexural rigidity is 5×10^{23} Nm (from Fjeldskaar and Pallesen, 1989).

Change in spreading rate

Ricard *et al.* (1988) have presented calculations of the effect of lateral variations in the upper mantle viscosity connected to diverging or converging continental masses. They conclude that large-scale geoid deflections will occur, with magnitudes depending on the plate velocities.

The large-scale geoid minimum deflection located at a diverging plate boundary, assuming a velocity of 10 cm yr^{-1}, could be 60 m. It is suggested that the Indian geoid low (or parts of it) could be explained in this way.

Thus it seems very unlikely that tectono-eustasy (with the exception of large-scale sedimentation) could operate without introducing large geoidal eustatic effects.

CONCLUSIONS

Eustatic sea-level changes are never globally uniform, because any cause of sea-level change simultaneously affects the Earth's geoid. Reconstruction of the ocean level is thus more complicated than is usually imagined. Eustatic changes have three main causes; (1) glacial eustasy, (2) tectono-eustasy and (3) geoidal eustasy.

This chapter has focused on the geoidal eustatic changes related to the deglaciation of the last great ice sheet in Europe, which has been shown to be a significant factor in sea-level changes. The results of our calculations indicate a fall in sea level in the vicinity of the former ice cap, caused by a gravitational change related to the deglaciation.

It has also been shown that any rapid change in the ocean water volume will affect the geoid. It is thus strongly suggested that rapid eustatic sea-level changes are never globally uniform.

Not even processes of more long-term eustatic change, such as variations of ocean ridge systems or small-scale sedimentation, seem to occur without affecting the geoid. Thus any eustatic curve claimed to be global may be questioned: rather, in the future more effort should be spent on constructing regional eustatic curves.

ACKNOWLEDGEMENTS

This work benefitted from the interest and enthusiasm of my colleagues at Rogaland Research Institute. The permission to reproduce copyright material (Fig. 1) was granted by the Editor, *Proceedings of the Royal Society of Victoria*. I thank the referee, Sierd Cloetingh, for his constructive comments.

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APPENDIX

For a spherical Earth, the geoid deformation $s(y)$ at a distance y from a point mass m is (Helmert, 1884):

$$s(y)=\frac{mR}{M}(R/y-1), \tag{1}$$

where R and M are the Earth's radius and mass, respectively.

For a circular mass anomaly of radius a and uniform thickness T, the geoid deformation (s_0) of the centre is (Helmert, 1884):

$$s_0=\frac{3\rho_1}{2\rho_0}\frac{aT}{R} \tag{2}$$

where ρ_1 and ρ_0 are the density of the mass anomaly and of the Earth, respectively.

The above approximations can be used for rough estimates of geoid deformation. For a more accurate estimate the load can be analysed into its harmonic components using the Fourier transform technique, which allows an aperiodic function (such as ice distribution) to be expressed as an integral sum over a continuous range of wavenumbers (the inverse of wavelengths).

For a harmonic load, the geoid deformation is given by the analytical expression (Cathles, 1975):

$$s(L)=\frac{4h(L)\pi G\rho_1 R}{(2L+1)g_0}, \tag{3}$$

where L is the order number of the load ($L=2\pi R/\lambda-\frac{1}{2}$); and $h(L)$ and ρ_1 are the amplitude and density of the load, respectively, G is the gravitational constant and g_0 is the surface gravity.