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Flexural rigidity of Fennoscandia inferred from the postglacial uplift

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Abstract. The Earth's response to glacial loading/unloading offers exceptional promise for the study of the physical properties of the lithosphere. In particular, tilting of paleoshorelines is very sensitive to the lithosphere rigidity. To determine the flexural rigidity, the isostatic response to deglaciation in Fennoscandia is modeled using an Earth model with a layered mantle viscosity overlain by an elastic lithosphere. The flexural rigidity and asthenosphere viscosity is allowed to vary to get a match between theoretical and observed present rate of uplift and tilting of paleoshorelines. Five different ice thickness models are used. For a relatively thin ice (2500 m in central areas) the resulting flexural rigidity is more or less uniform over Fennoscandia, with a value of 10^{23} N m (elastic thickness $t_e \approx 20$ km). This is regarded as minimum for the flexural rigidity of central Fennoscandia. The pattern of the present rate of uplift and the tilts of the paleoshorelines of the area also sets an upper bound of the flexural rigidity, 2.5 x 10^{25} N m ($t_e \approx 110$ km) in more central areas of Fennoscandia. The flexural rigidity at the western coast of Norway does not seem to exceed 10^{23} N m ($t_e \approx 50$ km). The most likely glacier model gives a flexural rigidity of 10^{23} N m ($t_e \approx 20$ km) at the Norwegian coast, increasing to above 10^{24} N m ($t_e \approx 50$ km) in central parts of Fennoscandia.

Introduction

The influence of the lithosphere in isostatic movements has been the subject of much debate. It is now accepted that the lithosphere behaves like an elastic plate overlying a viscous fluid. This concept is, however, not new. *Hertz* [1884] calculated the deflection of a thin elastic plate floating on the fluid substratum by a surface load. Later, Hertz' plate model was used to improve the understanding of flexure and its gravity effects [e.g., *Vening Meinesz*, 1931]. A historical review of the role of the lithosphere in isostatic models is given by *Wolf* [1993].

One of the best sources of information on the physical properties of the Earth's lithosphere and mantle is provided by data on postglacial uplift. Already in 1874, Nathaniel Shaler suggested that glacial depression could be used to compute the rigidity of the Earth's crust [*Shaler*, 1874]. The first estimates of the flexural rigidity of the lithosphere from postglacial uplift were, however, published first 90 years later by *McConnell* [1968] and *Walcott* [1970]. There is a general agreement about the importance of the lithosphere for the postglacial uplift, but little consensus about the thickness and rigidity of the lithosphere. *Niskanen* [1949] concluded at the end of a series of analyzes that lithospheric effects dominate the recent Fennoscandian uplift. *McConnell* [1968] found that the Fennoscandian data suggest a rigid upper mantle and crust down to about 120 km (flexural rigidity $D \approx 6 \times 10^{26}$ N m). Sea level observations from central Sweden and southern Finland analyzed by a three-layer Maxwell Earth model and a disk-load approximation of the deglaciation history of Fennoscandia has led *Wolf* [1987] to suggest lithosphere flexural rigidity less than 5 x 10^{24} N m (elastic thickness $t_e \approx 80$ km). On the basis of a three-dimensional viscoelastic model, data on the last glaciation in northern Europe, and data on the postglacial uplift in Fennoscandia, *Fjeldskaar and Cathles* [1991a] concluded the flexural rigidity to be equal to or less than 10^{24} N m ($t_e \approx 50$ km). Inversion of sea level observations from a site near the center of the Fennoscandian ice sheet and from three sites located beyond the margin of the former ice sheet gave a lithospheric thickness of 100-150 km [*Lambeck et al.*, 1990]. *Mitrovica and Peltier* [1993] found, with an inversion method, that the Fennoscandian uplift data suggest a lithospheric thickness ranging from 70 to 145 km. The purpose of the present paper is to constrain the estimate of the flexural rigidity for Fennoscandia by analysis of data on the present rate of uplift and tilts of paleoshorelines near the former ice sheet margin.

Lithosphere Flexure

If a load is applied to a fluid, the surface of the fluid will deform until the weight of the fluid displaced from the equilibrium level balances the applied load. If an elastic lithosphere covers the fluid and part of the applied load will be supported by the lithosphere and part by the buoyant forces of the fluid beneath acting through the lithosphere. Loads of short wavelength are supported by the lithosphere. The lithosphere thus acts as a low-pass filter. The characteristics of this filter depend on the elastic strength of the lithosphere. A measure of the elastic strength of the lithosphere is a parameter called the flexural rigidity, that is, the resistance to flexure. The flexural rigidity of the lithosphere is a function of the elastic thickness and is determined by the following equation:

$$D = \frac{E t_e^{\,3}}{12(1-\nu^2)}$$

where t_e is elastic thickness, ν is Poisson's ratio, and E is Young's modulus. E is determined as follows:

$$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}$$

where μ and λ are the Lamé parameters.

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Table 1. Modeling Parameters

Parameter	Value
Density of asthenosphere/ mantle	ρ=3300 kg m^{-3}
Density of glacier ice	ρ_{ice}=917 kg m^{-3}
Lamé parameters	λ=μ=3.34 x 10^{10} N m^{-2}
Reference rigidity	μ*= 10^{11} Nm^{-2}

For a flat Earth approximation with a uniformly thick elastic lithosphere the regional isostatic compensation can be calculated by the Fourier transform technique. The method used here is described in detail by *Cathles* [1975] and *Fjeldskaar and Cathles* [1991a]. The isostatic equilibrium displacements by flexure $F_O(k)$ due to a harmonic ice load $I(k)$ are achieved by subsidence:

$$F_0(k) = \frac{I(k)}{\rho g \alpha(k)}$$

where ρ is the density of the mantle (see Table 1), g is the gravity, k is the wave number, and for a compressible lithosphere $\alpha(k)$ is the "lithosphere filter" [*Cathles*, 1975] (1):

$$\alpha(k) = \left\{ \frac{2\mu^* k}{\rho \cdot g} \frac{(\lambda+\mu)}{(\lambda+2\mu)} \left[S^2 - k^2 t_e^2 \right] + \left[CS + k t_e \right] \right\} \Big/ \left[S + k t_e C \right]$$

where λ and μ are Lamé parameters, μ^* is the reference rigidity, S is sinh kt_e, C is cosh kt_e. (Please note a typing error in the formula of *Fjeldskaar and Cathles*, [1991a]).

The elastic lithosphere will speed up the rate of compensation. The subsidence as a function of time (t) is (2):

$$F(k, t) = F_O(k) e^{-t\, \alpha(k)/\tau}$$

where τ is the relaxation time for the viscous fluid mantle below the lithosphere. The method for calculating the relaxation time is described in *Cathles* [1975] and *Fjeldskaar and Cathles* [1991a].

"The lithosphere filter" $\alpha(k)$ is alternatively approximated by the formula [*Nadai*, 1950] (3):

$$\alpha(k) = 1 + \frac{k^4}{\rho g} D(k)$$

Equations (1) and (3) describe a filter that is applied to the sublithospheric Earth model response, both apply to thin plates (kt_e small, which is the case for the Fennoscandian postglacial uplift). The differences between "thin" and "thick" elastic plates are discussed by *Comer* [1983] and *Wolf* [1985]. In the modeling results reported here, we use the lithospheric filter as defined in equation (3). The calculated isostatic equilibrium deflections by this filtering technique are similar to the deflections calculated for an isotropically elastic, uniformly thin, spherical shell [*Brotchie and Silvester*, 1969]. Equilibrium deflections under a uniform circular load of different radius for the two methods (for a flexural rigidity of 3.75 x 10^{23} N m) are shown in Figure 1.

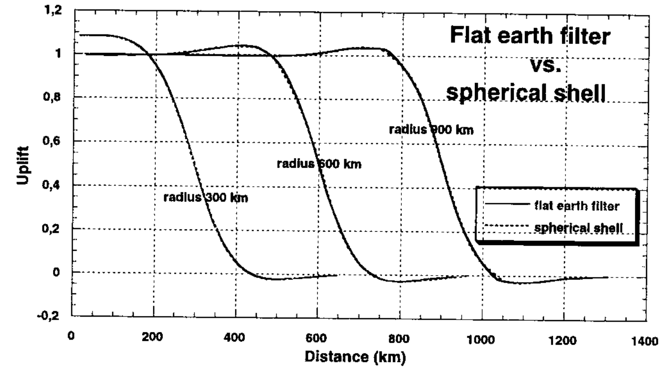

Figure 1. Equilibrium isostatic deflections under uniform circular loads of different radius for the flat Earth filter method (the radius consisting of 10 grid points, solid lines) and for the spherical shell solution of *Brotchie and Silvester* [1969] (dotted lines). The flexural rigidity is 3.75 x 10^{23} N m.

The filtering approach ignores the coupling between the lithosphere and mantle. To quantify the errors introduced by neglecting full coupling, we have done some calculations to be compared with *McConnell's* [1968] relaxation spectra for a fully coupled Earth model. The relaxation spectra for a mantle of viscosity 0.6 x 10^{22} Pa s overlain by a 140 km thick lithosphere with a rigidity of 9.6 x 10^{10} N m^{-2} (corresponding to McConnell's model 62-10) is shown in Figure 2. The spectra for the "uncoupled" solution deviates in relaxation time by maximum 500 years from McConnell's fully coupled solution. However, a small decrease of the flexural rigidity (from 585 to 450 x 10^{23} N m, equaling a decrease of t_e from 140 to 128 km) will correct the differences (Figure 2). There are, however, still some differences for short wavelengths. However, wavelengths shorter than 1000 km are not a significant contributor to the final shoreline tilt (Figure 3); a maximum of 10% of the tilt can be explained by wavelengths shorter than 1000 km. The differences between the relaxation spectra for a fully coupled model compared to the filtering method will consequently be systematic and small (in the sense that a very small change in the flexural rigidity would correct them).

The reason that wavelengths shorter than 1000 km are not a significant contributor to the final shoreline tilt (Figure 3) is

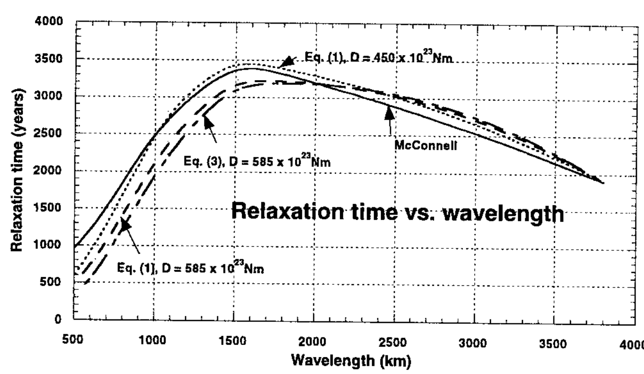

Figure 2. Relaxation spectra for a fully coupled lithosphere [*McConnell*, 1968] compared to our lithosphere filter solution $\tau/\alpha(k)$ (equations (1) and (3)).

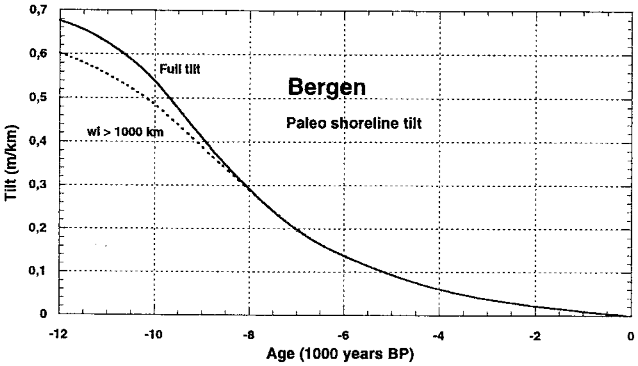

Figure 3. Paleoshoreline tilt for Bergen area calculated with a flexural rigidity of 500 x 10^{23} Nm (solid line). The dotted line shows the contribution to the calculated tilt of wavelengths greater than 1000 km. The contribution is more than 90%.

partly that these wavelengths do not contribute too much to the load spectrum and partly that these wavelengths are attenuated by the elastic lithosphere. A thinner lithosphere will act as a less effective filter; a higher percentage of the total amplitude spectrum will be of short wavelengths. Thus, with a thinner lithosphere, the relaxation spectra (Figure 2) will be modified. However, the conclusions given above are also valid for a thinner lithosphere. Thus the filtering method introduces some errors, but the errors are scarcely of practical significance for the glacial uplift.

Postglacial Uplift

The postglacial uplift has been mapped by several methods. This study uses two types of uplift data: (1) present rate of uplift and (2) shoreline tilts versus time. These observations are mainly results of the movements of the solid Earth; they are scarcely affected by movements of the sea level. The movements of the solid Earth are here assumed to have glacial isostatic origin connected to the melting of the last ice sheets (geoid movements are neglected).

Present Rate of Uplift

The observed present rate of uplift in Scandinavia relative to mean sea level increases from 0 mm/yr at the western coast of Norway to 9 mm/yr in central parts of Sweden (Figure 4). To obtain the uplift of the crust relative to the geoid, the uplift rate has to be corrected for the eustatic sea level changes, which would, probably, add approximately 1 mm to the numbers given in Figure 4. However, there is no absolute measurement of the present eustatic sea level rise. The closest we can come to an estimate is probably analyzes of all tide gauge data in the world, which show that the eustatic rise, reflecting a change in the water volume, is estimated as 1.15 ± 0.38 mm/yr [*Nakiboglu and Lambeck*, 1991]. The maximum present uplift rate is thus close to 10 mm/yr in central parts.

Shoreline Tilts Versus Time

Shoreline diagrams give the observed shoreline tilting versus time. Such curves are available or may be constructed from local shore level displacement curves for a number of localities in Fennoscandia, of which six are selected here (For locations see Figure 4).

The accuracy of the shore level displacement data, on which the gradient curves are based, varies because different determination methods have been used. The curves from southern Norway, Sweden, and Finland have the highest accuracy, because they are based on dated cores from dammed lakes of different heights. The uncertainties could have three different sources [*Kjemperud*, 1986]: (1) determination of the isolation contact, (2) determination of the threshold height, and (3) dating of the sediments. The uncertainty for source 1 is 1-2 cm and for source 2 is less than 1 m. The most crucial point is the dating of the sediments. The uncertainty in radiocarbon dating is generally less than 200-300 years [*Kjemperud*, 1986]. The total uncertainty in the determination of the late glacial and postglacial gradients is thus probably less than 0.15 m/km. The curves from northern Norway (Altafjord and Varangerfjord) are exceptions; they are based on geomorphological data. The largest uncertainty in these curves is the age determination, because of lack of direct dating material of the shoreline. We have here assumed the total uncertainty to be 750 C 14 years.

Viscosity Structure

The modeled tilting of paleoshorelines at particular locations peripheral to the former ice load and the pattern of present rate of uplift consistent with the observations strongly suggest a low-viscosity asthenosphere. This has previously been reported by this author in several publications. Models with lower mantle viscosity 1.0 - 2.0 x 10^{21} Pa s, upper mantle viscosity 0.7 - 1.0 x 10^{21} Pa s, and a low-viscosity asthenosphere of thickness less than 150 km (viscosity less than 7.0 x 10^{19} Pa s) fit equally well

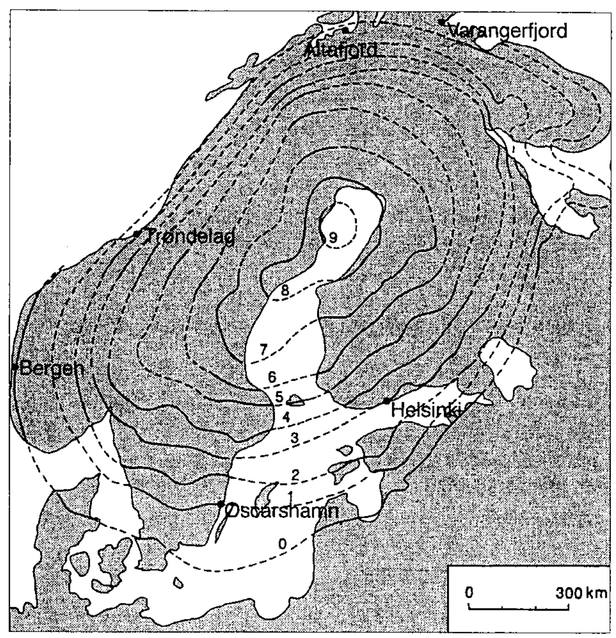

Figure 4. Observed apparent rate of uplift in Fennoscandia [*Ekman*, 1989]. Solid lines indicate areas covered by observations.

with the observations [*Fjeldskaar*, 1994]. It was pointed out [*Fjeldskaar and Cathles*, 1991b] that the present rate of uplift is scarcely affected by the lithosphere rigidity; the uplift pattern calculated when using a rigidity of 10^{23} or 10^{25} N m is hardly distinguishable. Paleoshoreline tilts suggest the flexural rigidity of the lithosphere to be significantly lower than 5 x 10^{24} N m (t_e < 90 km; [*Fjeldskaar and Cathles*, 1991a]).

Glacial Isostasy and Hydroisostasy

Glacial Isostasy

The movements of the solid Earth to reestablish isostatic equilibrium during changes of the ice loads are calculated by (2) based on the deglaciation history described in the section on deglaciation data. Previous results (compare section on viscosity structure) were obtained by using only one ice load model (basically the same as termed ice model 3; see below). The ice thickness, however, is uncertain. The modeled flexural rigidity is highly sensitive to the ice model thickness. Therefore in the present paper, which is focused on modeling the flexural rigidity, several models of the ice thickness are run.

One of the main conclusions of previous works was the necessity of a low-viscosity asthenosphere. This conclusion is not altered, no matter what ice load model is used. However, to run the different ice thickness models, the viscosity structure of the mantle needs small modifications (compared to previous results) in each case, because one of the basic assumption in the modeling is a present-day rate of uplift in the central Baltic Sea of approximately 10 mm/yr.

The previous work suggested a range of possible lower mantle, upper mantle, and asthenospheric viscosities. The resulting uplift difference within this range (given above) was very small. We have therefore in the present study chosen to run the different ice thickness models with a lower and upper mantle viscosity of 10^{21} Pa s and basically an asthenospheric thickness of 75 km. The asthenospheric viscosity will, as mentioned above, be slightly different for each ice model.

Hydroisostasy

The movement of the ocean bottom caused by the sea level change is calculated separately by (2), using the same viscosity structure as for the glacial isostasy. The land-ocean distribution during the deglaciation is assumed to be as at present. The late glacial and postglacial sea level change, assumed to take place outside the present land area (Figure 5), is in accordance with published eustatic curves for late glacial and postglacial time [e.g. *Shepard*, 1963]. As pointed out by *Johnston* [1993], the

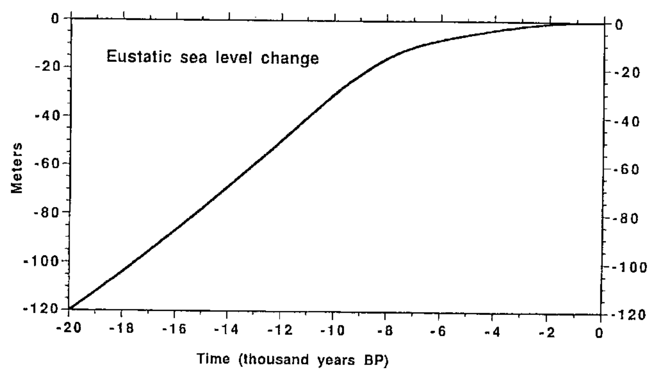

Figure 5. The eustatic sea level curve used in the calculations.

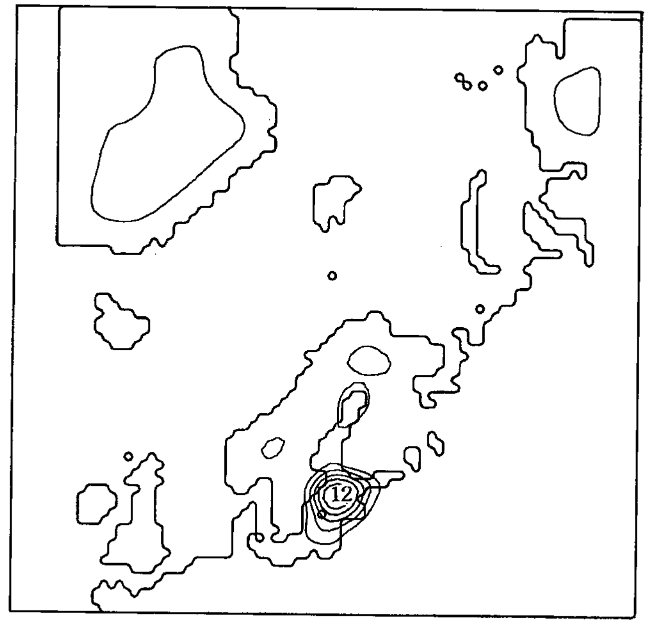

Figure 6. The theoretical movement of the solid Earth in response to the water retreat in the Baltic Sea at 9000 years B.P., calculated with a mantle viscosity of 1.0 x 10^{21} Pa s, a 75 km low-viscosity asthenosphere of viscosity 1.3 x 10^{19} Pa s, and a flexural rigidity of 10^{24} N m. Contour interval is 2 m.

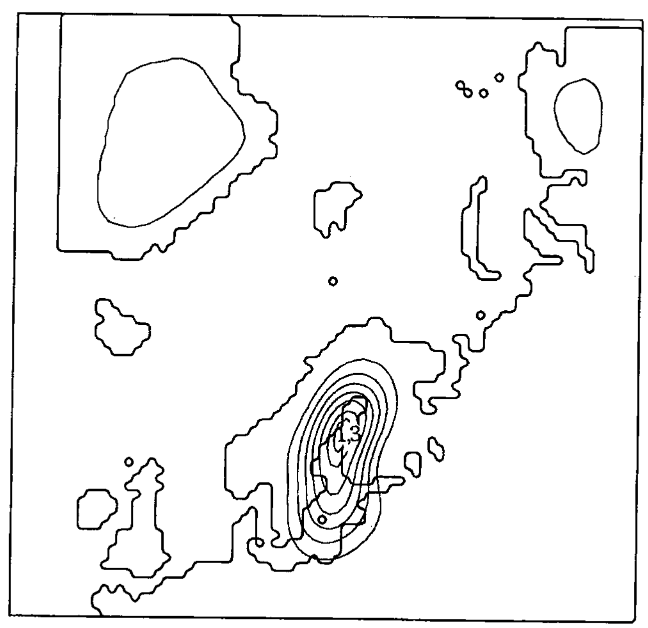

Figure 7. The effect of the water retreat in the Baltic Sea on the present rate of uplift, calculated with a mantle viscosity of 1.0 x 10^{21} Pa s, a 75 km low-viscosity asthenosphere of viscosity 1.3 x 10^{19} Pa s, and a flexural rigidity of 10^{24} N m. Contour interval is 0.2 mm/yr.

modeling of hydroisostasy based on published eustatic curves is inadequate for sites in the Baltic Sea where the load is of opposite sign to "global" sea level change. The sea level in the Baltic Sea has fallen as a consequence of the postglacial uplift. Assuming a flexural rigidity of 10^{24} N m ($t_e \approx$ 50 km) and an asthenospheric viscosity of 1.3 x 10^{19} Pa s, the theoretical effect of this water retreat is up to 12 m in the southern Baltic Sea at 9000 years B.P. (Figure 6). From 9000 to 7000 years B.P. the maximum is moving north because of the ice retreat [cf. *Eronen*, 1983], with a maximum of 14 m. The effect of the water retreat on the present rate of uplift is a maximum of 1.3 mm/yr (Figure 7). These values, though significant for the uplift in the Baltic Sea, will not change the overall uplift pattern and the conclusions of our previous work, because of its local nature.

Deglaciation Data

The deglaciation of the last ice age is relatively well established by observations of marginal moraines. The deglaciation history used here (Figure 8) was compiled by B.G. Andersen, presented by *Denton and Hughes* [1981]. It is slightly modified by new data from Svalbard [*Elverhøi et al.*, 1993] and south Sweden [*Björck*, 1989].

The deglaciation model has been digitized with a spatial resolution of approximately 50 km by 50 km. The melting rate between two ice sheet configurations (Figure 8) is assumed be constant. The area is assumed to have been ice free at 8500 years B.P.

Glacial thicknesses, however, are uncertain based on a paucity of direct geological evidence. Because of the uncertainty of the glacial thickness, the isostatic modeling is done for five ice thickness models, including the one of Figure 8 (termed model 3; see below). There are, however, some indications of ice thickness. These data come from the former ice front.

Ice Front Thickness

In previous studies [e.g., *Fjeldskaar*, 1994] we have used a rather thin glacier front, as a mean value over the mountains of western Norway (Figure 9). In the fiords of western and northern Norway, however, there are observations of the ice front thickness of the Younger Dryas age (11,000 - 10,000 years B.P.), when the outermost part of the Norwegian coast was ice free. The observed ice thickness in the Hardangerfiord area (western Norway) of Younger Dryas age [*Follestad*, 1972] is 1000 m approximately 40 km from the ice margin (Figure 9). These observations represent only one stadium in the deglaciation. However, this stadium is the most important for the development of the late glacial shoreline tilts. One must, however, realize that the ice front thickness is measured in the fiords and relative to sea level. In the modeling this ice front thickness is used as a mean value over the entire area (e.g., for western Norway). Thus it must be regarded as an absolute maximum ice front thickness. The influence of the peripheral ice thickness on the shoreline tilts of Bergen area is, however, significant only if the flexural rigidity is significantly less than 10^{24} N m ($t_e \approx$ 50 km, Figure 10). The Younger Dryas glacier front ice thickness used for the various regions (ice model 3; see below) in this paper is shown in Figure 11.

Ice Thickness Models

Five ice thickness models have been run: our preferred and most likely model (termed model 3, Figure 8), two thinner ice models (75% of the ice thicknesses of model 3 (model 1) and 85% of the ice thicknesses of model 3 (model 2)), and two thicker ice models; model 4 (25% thicker ice than model 3) and model 5 (50% thicker ice than model 3).

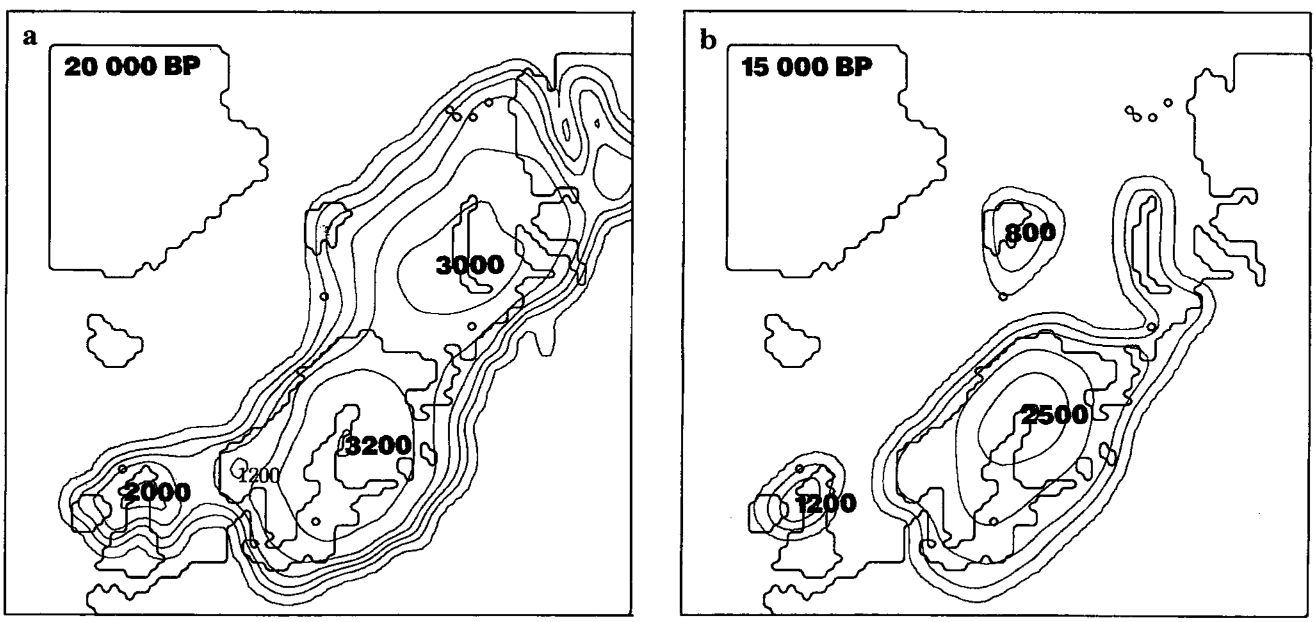

Figure 8. The extent and thickness of the ice sheet during the deglaciation in Northern Europe, partly based on *Denton and Hughes* [1981]: (a) 20,000 years B.P., (b) 15,000 years B.P., (c) 11,500 years B.P., (d) 10,500 years B.P., and (e) 9,300 years B.P. Contour interval is 500 m, except for (e), where the contour interval is 200 m.

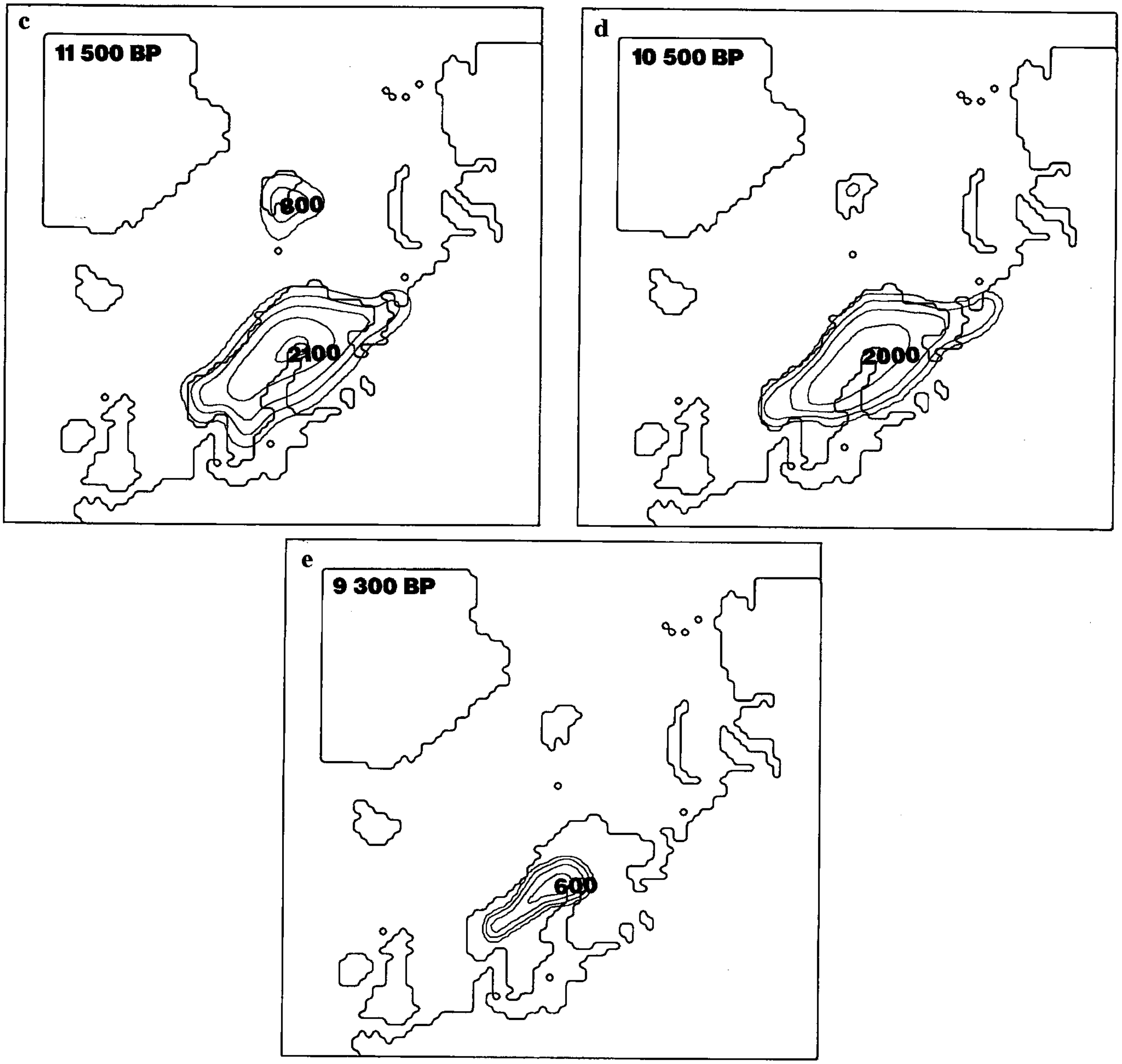

Figure 8. (continued)

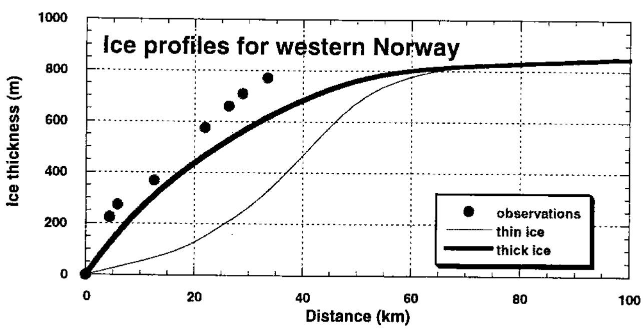

Figure 9. Theoretical versus observed ice front profiles of Younger Dryas age (11,000-10,000 years B.P.) for western Norway.

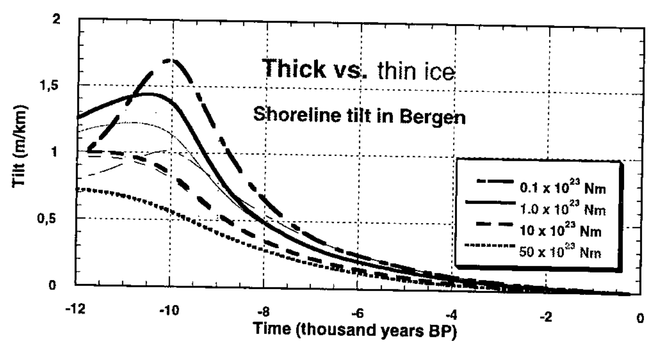

Figure 10. The influence of thick versus thin Younger Dryas ice front thickness on the paleoshoreline tilt in Bergen area. Thick lines show resulting tilt (assuming different flexural rigidities) for the thick glacier ice front. The thin lines show results for the thin ice front.

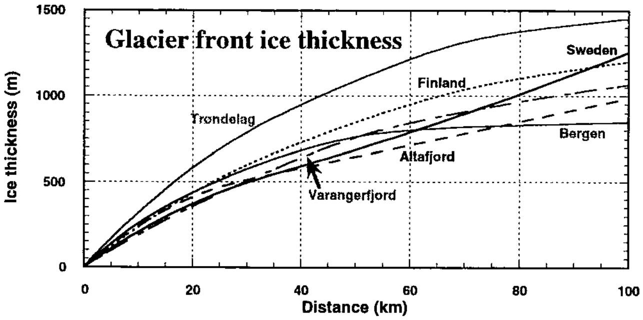

Figure 11. The glacier front thickness of Younger Dryas age for various regions used in the calculations (ice model 3).

Present Rate of Uplift

With the assumption of a present-day rate of uplift in central Baltic Sea of approximately 10 mm/yr, the asthenospheric viscosity will be different for the five ice models. When the Baltic Sea water retreat is taken into account, model 1 gives an asthenospheric viscosity of 2.3 x 10^{19} Pa s, model 2 gives a viscosity of 1.7 x 10^{19} Pa s, model 3 gives a viscosity of 1.3 x 10^{19} Pa s, model 4 gives a viscosity of 1.0 x 10^{19} Pa s and model 5 gives a viscosity of 0.85 x 10^{19} Pa s. The models of the asthenospheric viscosity are based on an asthenospheric thickness of 75 km and a flexural rigidity of 10^{24} N m ($t_e \approx 50$ km).

The pattern of present rate of uplift is not drastically different for the five models. However, it is clear that the area presently under upheaval is increasing for increasing ice thickness under the assumption of a central uplift rate of approximately 10 mm/yr (Figures 12a-12e). The minimum model gives uplift values somewhat below what is observed. However, the fit would be better if the eustatic sea level rise is reduced below 1 mm/yr. There is actually room for a somewhat lower sea level rise in the results from *Nakiboglu and Lambeck* [1991]. Anyway, from the pattern of the theoretical present rate of uplift it is reasonable to suggest that our model 1 is in the lowermost range of what is a reasonable ice thickness model.

Models 2, 3, and 4 are within the acceptable range of variation from the observed present rate of uplift (Figures 12a-12d), especially with the uncertainty of the present eustatic rise in mind (1.15 ± 0.38 mm/yr). With an increase of the ice thickness beyond model 4, the misfit (in particular in middle and northern Norway) is increasing to unacceptable values. Model 5 gives a present-day uplift (Figure 12e) in middle and northern Norway which is more than 1 mm/yr higher than observed and a much more circular pattern than is observed.

However, this problem may be overcome by increasing the asthenosphere thickness and thereby increasing the relaxation time for short wavelengths. If the asthenosphere thickness is increased to 250 km, the viscosity of the asthenosphere will be 1.2 x 10^{20} Pa s. The pattern of the present rate of uplift will change to a more elliptical shape (Figure 13) with values close to the observed present rate of uplift. There is, however, a trade-off between the viscosity and the flexural rigidity, so, if the flexural rigidity is increased to, say, 500 x 10^{23} N m ($t_e \approx 200$ km), the pattern of the present rate of uplift again gets more circular (Figure 14).

On the basis of the present rate of uplift alone it seems to be a reasonable conclusion that the possible ice thickness could, at least, vary from 85% to 150% of the suggested value of *Denton*

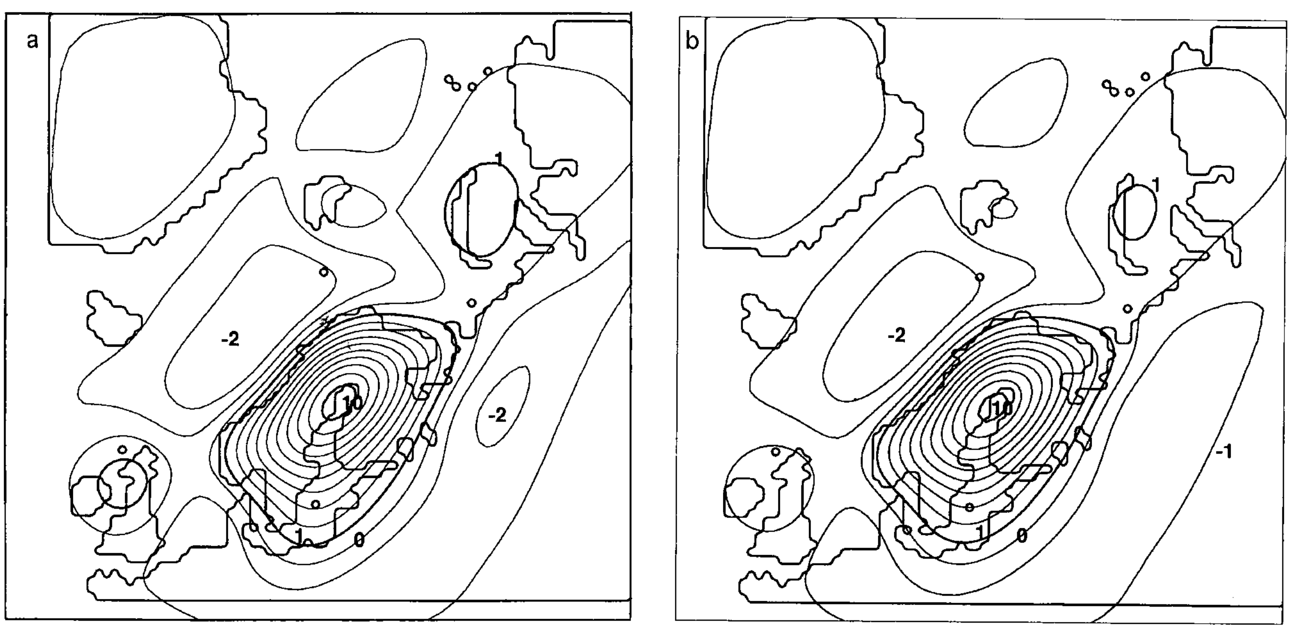

Figure 12. Pattern of theoretical present rate of uplift. (a) ice model 1, assuming an asthenosphere viscosity of 2.3 x 10^{19} Pa s. (b) Ice model 2, assuming an asthenosphere viscosity of 1.7 x 10^{19} Pa s. (c) Ice model 3, assuming a viscosity of 1.3 x 10^{19} Pa s. (d) Ice model 4, assuming a viscosity of 1.0 x 10^{19} Pa s. (e) ice model 5, assuming a viscosity of 0.85 x 10^{19} Pa s. The models of the asthenospheric viscosity are based on a flexural rigidity of 10^{24} N m. Please note that the 1 mm/yr contour line (the heavy solid line) corresponds to the zero contour line of the observed apparent rate of uplift (Figure 4). Contour interval is 1 mm/yr.

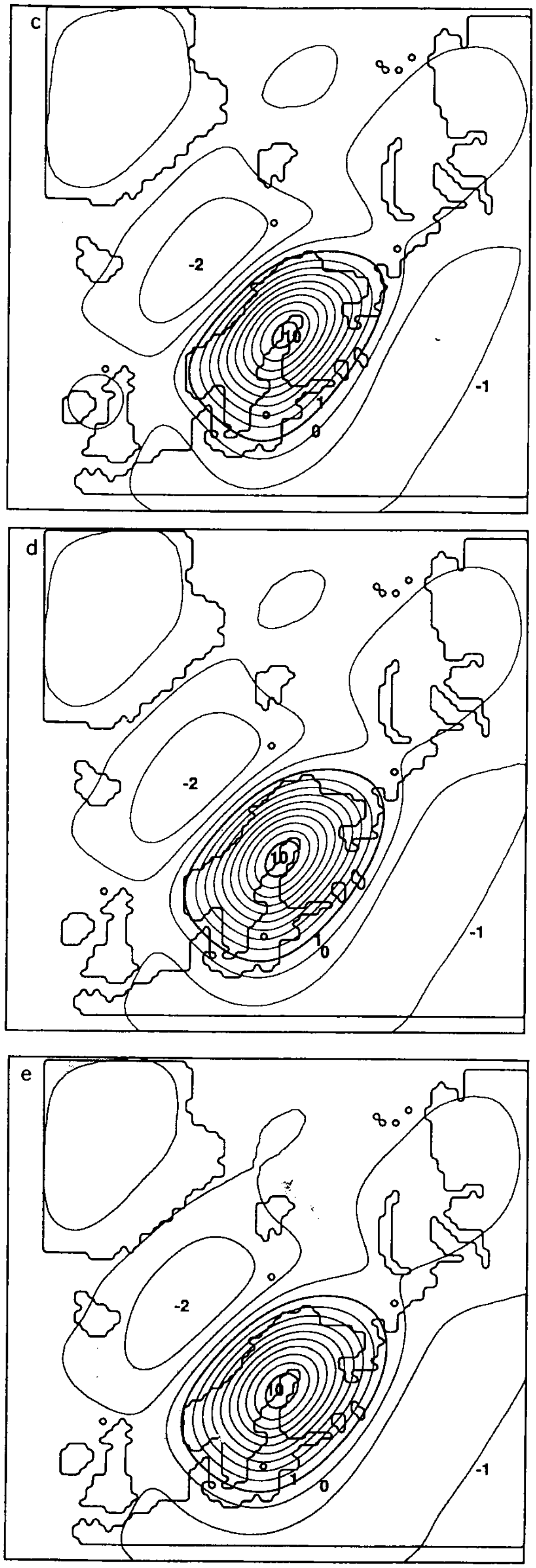

Figure 12. (continued)

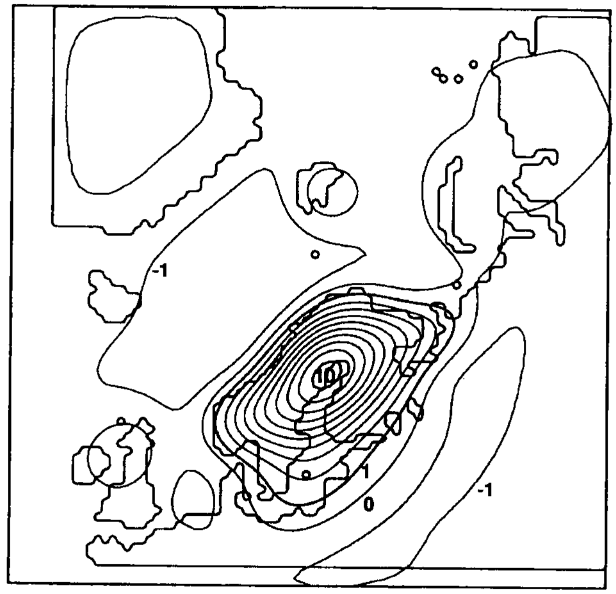

Figure 13. Pattern of theoretical present rate of uplift. Ice model 5 is used, with an asthenosphere of thickness 250 km of viscosity 0.12 x 10^{21} Pa s and a flexural rigidity of 10^{24} N m. Please note that the 1 mm/yr contour line (the heavy solid line) corresponds to the zero contour line of the observed apparent rate of uplift (Figure 4). Contour interval is 1 mm/yr.

and Hughes [1981] or a possible maximum ice thickness range from 2500 to 4500 m. The thickest ice model is only an option if the asthenosphere is thick and the flexural rigidity is relatively low, significantly less than 10^{26} N m ($t_e \approx 250$ km; the paleoshoreline tilts, however, show that this option is not viable; see below).

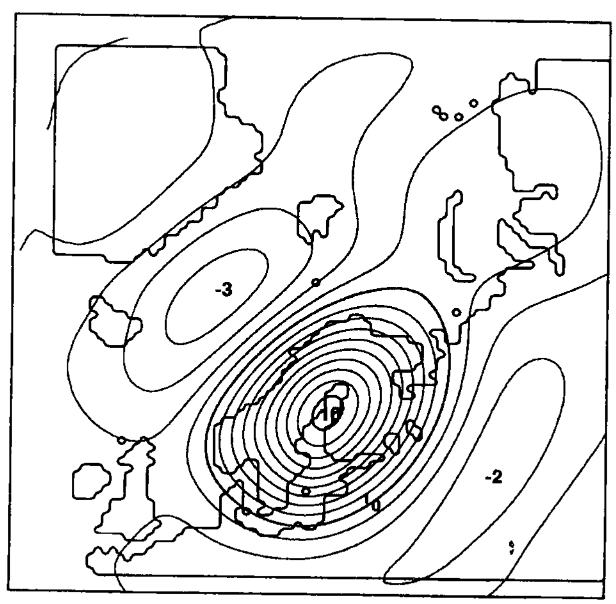

Figure 14. Pattern of theoretical present rate of uplift. Ice model 5 is used, with an asthenosphere of thickness 250 km of viscosity 0.43 x 10^{21} Pa s and a flexural rigidity of 10^{26} Nm. Please note that the 1 mm/yr contour line (the heavy solid line) corresponds to the zero contour line of the observed apparent rate of uplift (Figure 4). Contour interval is 1 mm/yr.

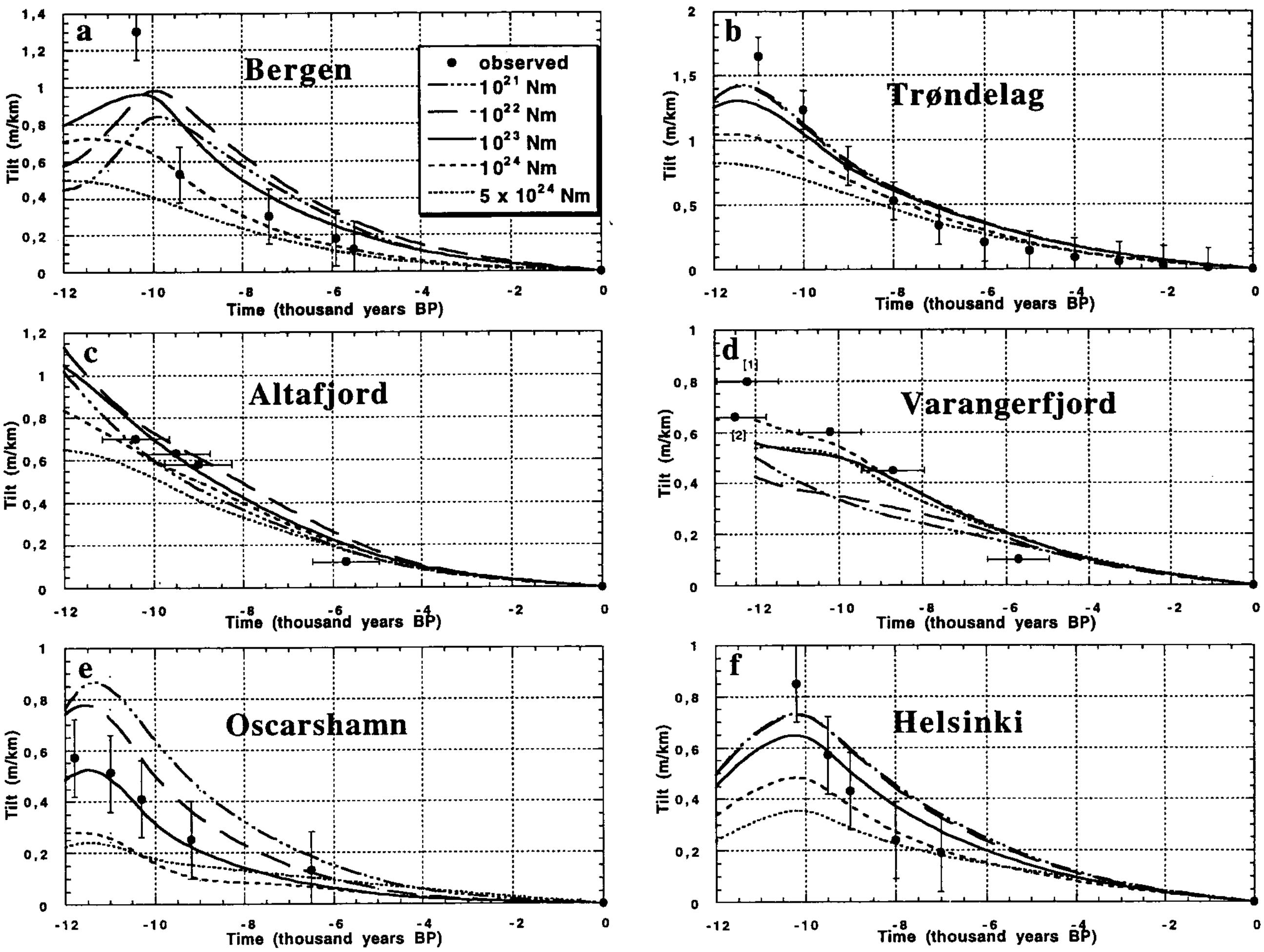

Figure 15. Theoretical and observed shoreline tilting versus time for ice model 1. (a) Bergen [*Kaland*, 1984], (b) Trøndelag [*Kjemperud,* 1986], (c) Altafjord [*Marthinussen*, 1960, 1962], (d) Varangerfjord area ([1] high gradient at 12 000 BP from *Sollid et al.*, [1973]; [2] low gradient from *Marthinussen,* [1974]), (e) Oscarshamn, Sweden [*Svensson*, 1991], and (f) Helsinki, Finland [*Eronen*, 1983; *Eronen et al.*, 1993]. The theoretical values are based on uniform flexural rigidity of 10^{21} N m, 10^{22} N m, 10^{23} N m, 10^{24} N m and 5 x 10^{24} N m, respectively. Locations are shown in Figure 4.

Paleoshoreline Gradients

Four sites along the coast of Norway and two sites in central Scandinavia with observed paleoshoreline tilts have been selected for the present investigation (Figure 4). The important part of the paleoshoreline curve for the determination of the flexural rigidity is the late glacial part (prior to 8000 years B.P.). It is difficult to discriminate between different flexural rigidity models based on the postglacial part of the curve alone [*Fjeldskaar and Cathles*, 1991a]. The tilting of the paleoshorelines shows the following.

Ice model 1. No matter what flexural rigidity that is chosen, it seems impossible to mimic the high late glacial shoreline gradients of the Bergen and Trøndelag (or the Finland) areas (Figure 15). As was shown by *Fjeldskaar* [1994], this problem cannot be overcome with a modified viscosity profile. It is thus strongly suggested that the ice was significantly thicker than ice model 1.

Ice model 5. For an ice model 150% of *Denton and Hughes'* [1981] ice thickness model the high late glacial tilts of Bergen and Trøndelag areas could be mimicked with a thin lithosphere (1-10 x 10^{24} N m, Figure 16). However, a thin lithosphere gives too high tilts in Altafjord, Varangerfjord, Sweden, and Finland. To get a better fit in those areas, it is necessary to increase the flexural rigidity to 10^{26} N m ($t_e \approx 250$ km). The theoretical present rate of uplift based on this high value of flexural rigidity will be significantly higher than observed, in particular, in northern Norway (Figure 14). It is therefore suggested that the ice model 5 is unreasonably thick.

Ice model 2. Ice model 2 (85% of the thickness of model 3) gives a closely uniform flexural rigidity of 10^{23} N m ($t_e \approx 20$ km) on all localities (Figure 17).

Ice model 3. For the ice model of Figure 8 the flexural rigidity that gives the best fit seems to be changing slightly from one location to the other (Figure 18). Data from Altafjord do not resolve the flexural rigidity at all (Figure 18c). For Varangerfjord

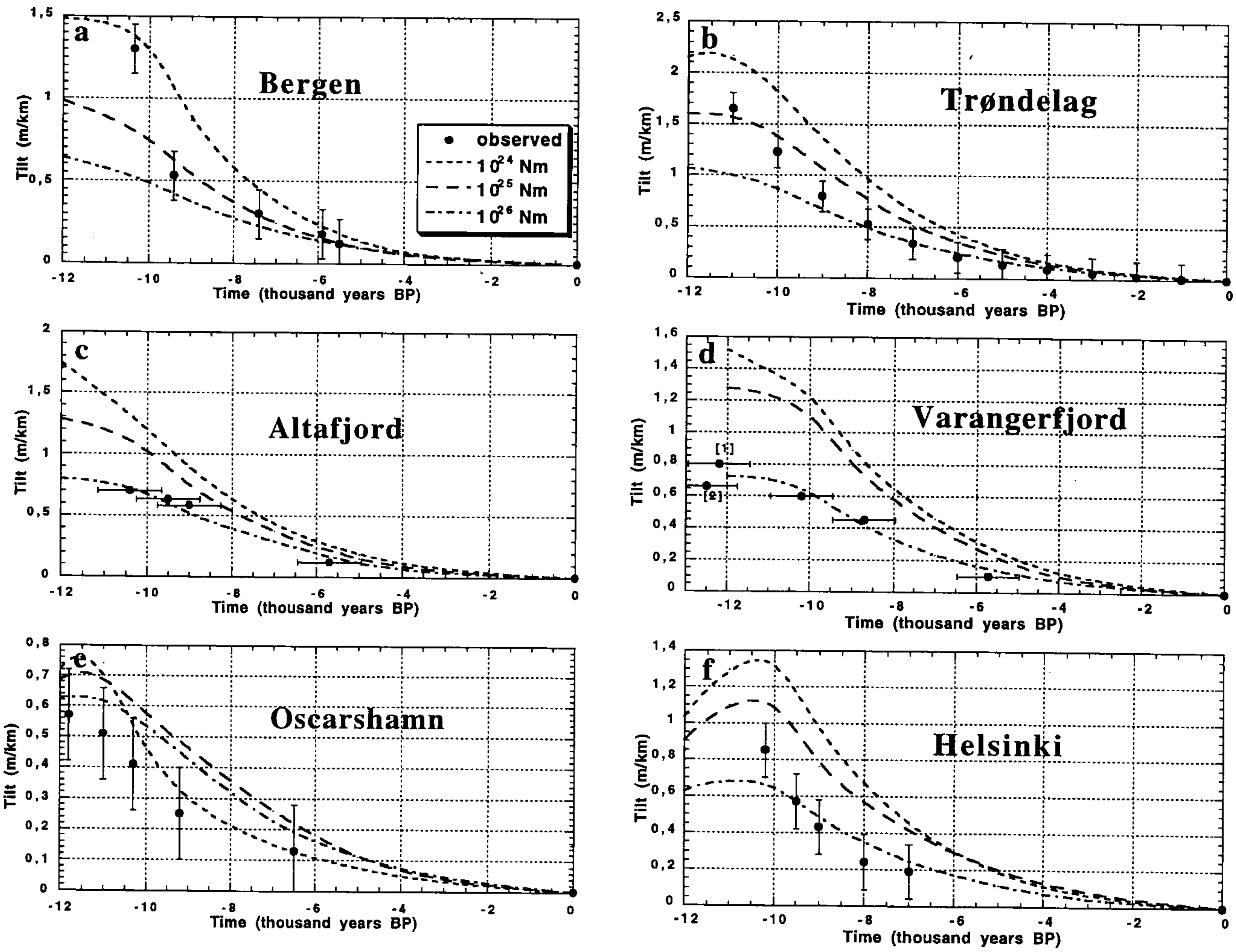

Figure 16. Theoretical and observed shoreline tilting versus time for ice model 5 (compare text to Figure 15). The theoretical values are based on uniform flexural rigidity of 10^{24} N m, 10^{25} N m and 10^{26} N m.

the flexural rigidity is obtained close to 10^{23} N m (Figure 18d). In Bergen and Trøndelag the flexural rigidity seems to be somewhat higher, between 10^{23} and 10^{24} N m (Figure 18a, b). The same is the case for Oscarshamn, southern Sweden (Figure 18e). Further inland, in the Helsinki area, the flexural rigidity is above 10^{24} N m (Figure 18f). Thus there seems to be an increasing flexural rigidity toward the east, exactly how is difficult to say without doing calculations with models that enable lateral changes in flexural rigidity.

Ice model 4. For the ice model 4 (125% the thickness of model 3) the picture is somewhat unclear, but, in general, the results indicate a higher flexural rigidity than for model 3. For

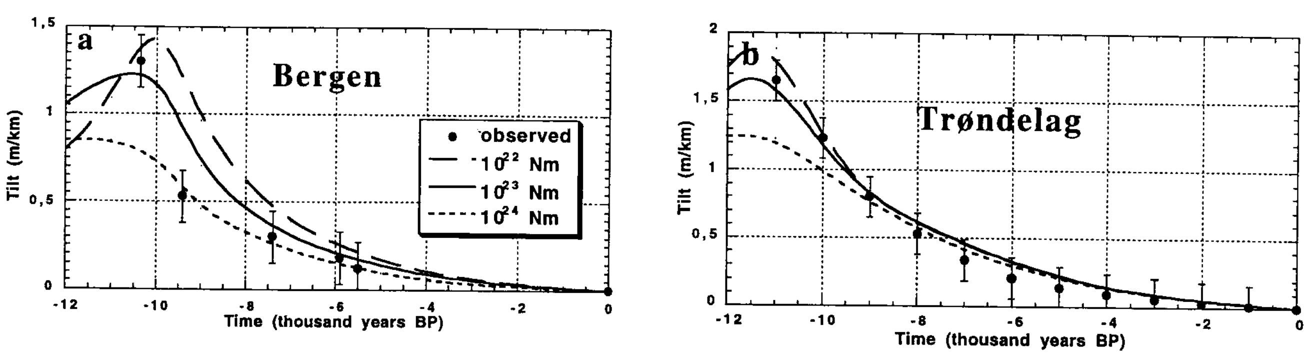

Figure 17. Theoretical and observed shoreline tilting versus time for ice model 2 (compare text to Figure 15). The theoretical values are based on uniform flexural rigidity of 10^{22} N m, 10^{23} N m and 10^{24} N m.

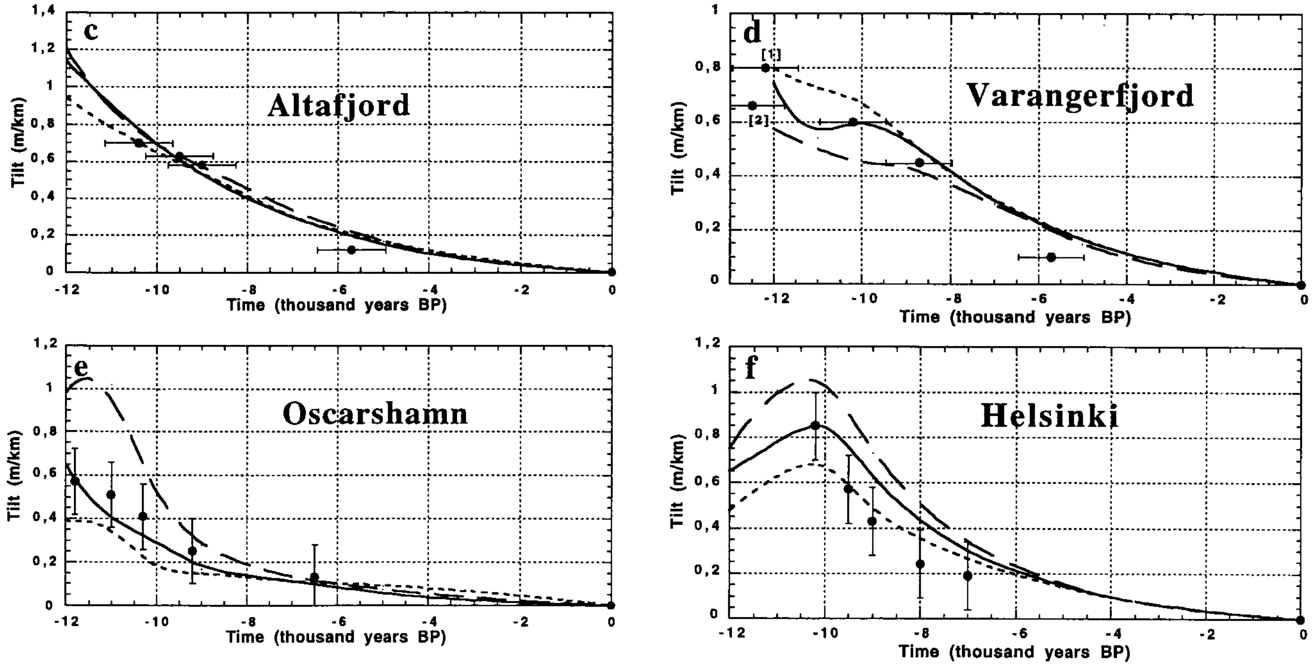

Figure 17. (continued)

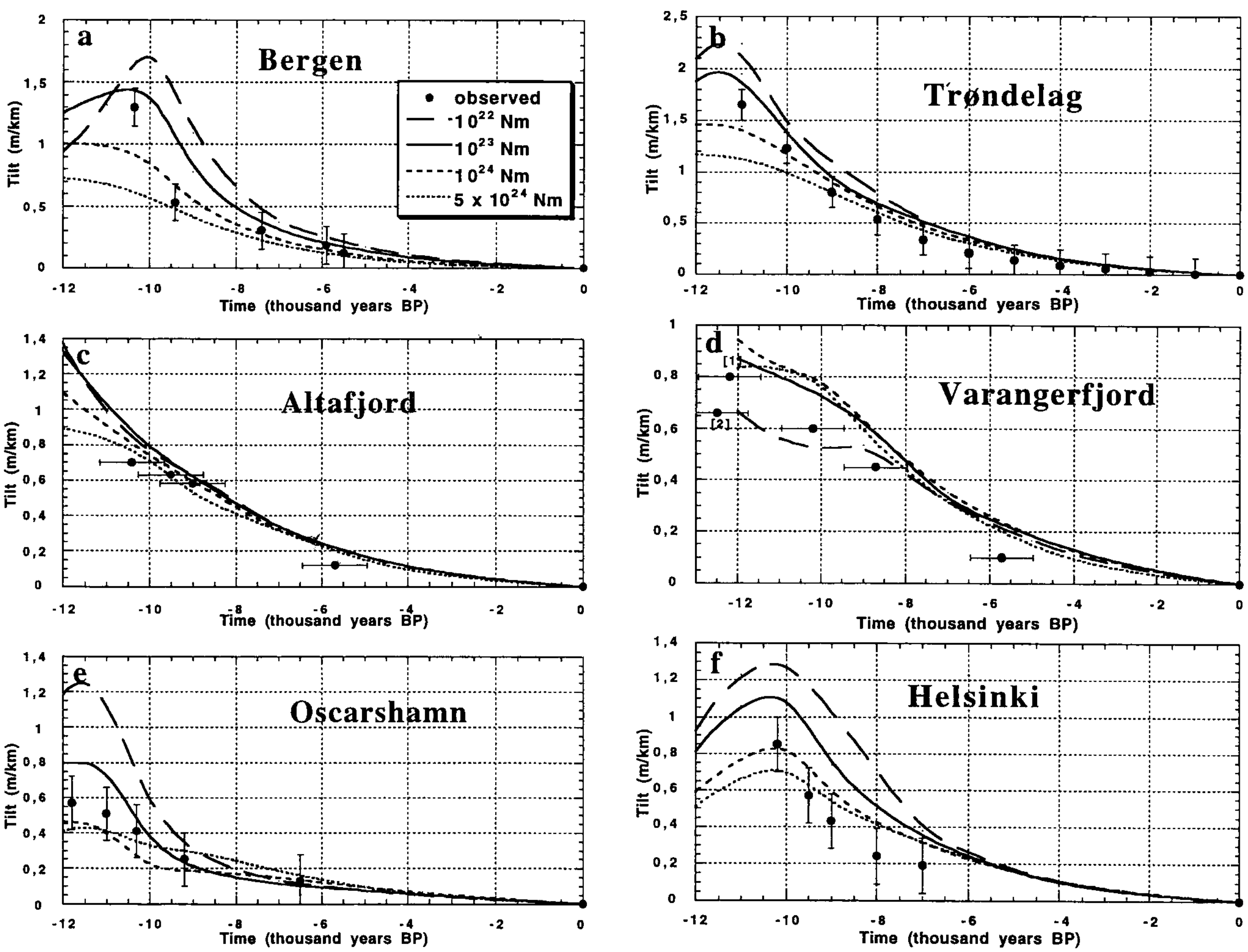

Figure 18. Theoretical and observed shoreline tilting versus time for ice model 3 (compare text to Figure 15). The theoretical values are based on uniform flexural rigidity of 10^{22} N m, 10^{23} N m, 10^{24} N m and 5×10^{24} N m.

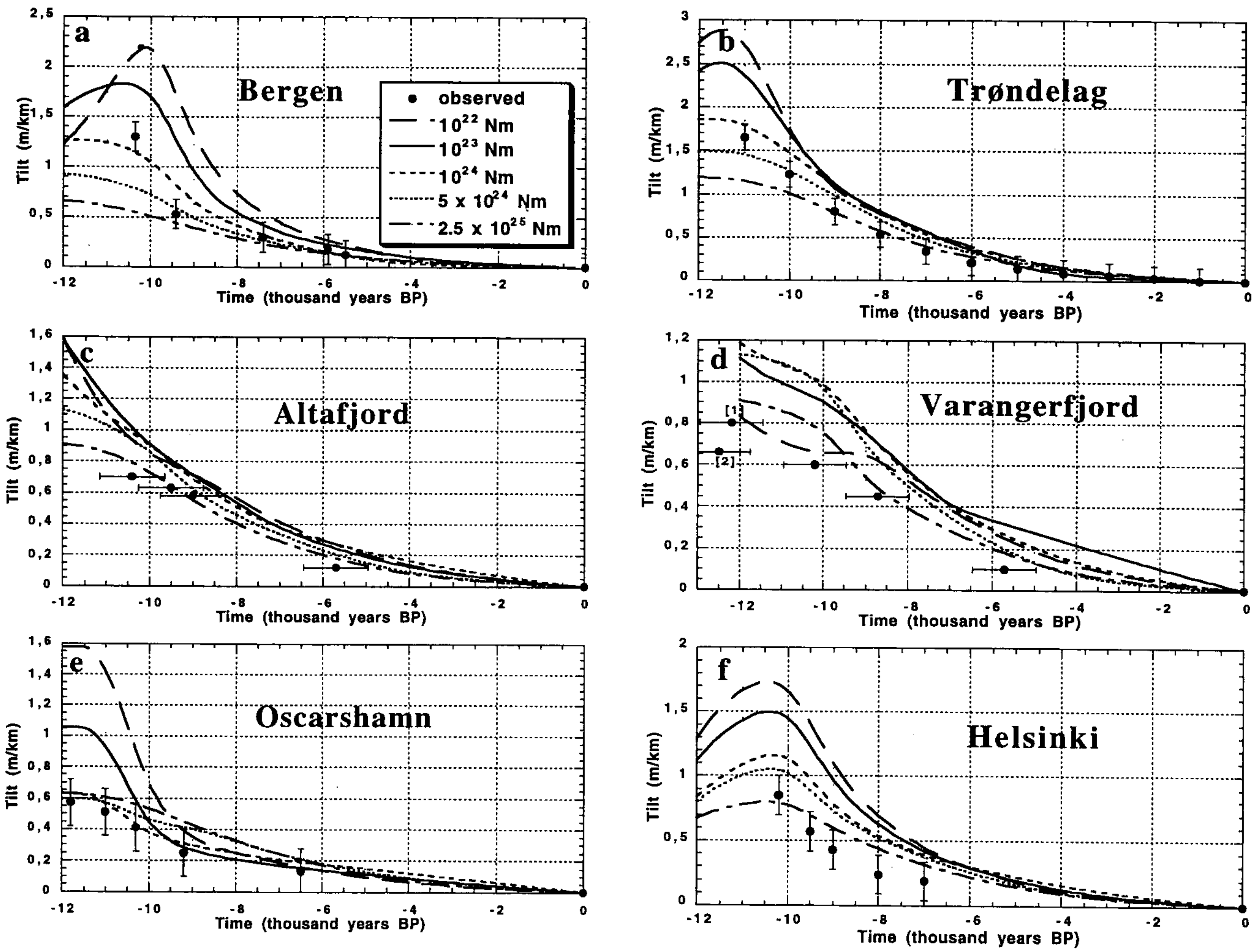

Figure 19. Theoretical and observed shoreline tilting versus time for ice model 4 (compare text to Figure 15). The theoretical values are based on uniform flexural rigidity of 10^{22} N m, 10^{23} N m, 10^{24} N m, 5 x 10^{24} N m and 2.5 x 10^{25} N m.

western Norway the resulting flexural rigidity is close to 10^{24} N m (Figures 19a and 19b); Bergen is somewhat below, and Trøndelag is somewhat above 10^{24} N m. For the locations in northern Norway the results seem to indicate a very high flexural rigidity (close to 2.5 x 10^{25} N m ($t_e \approx$ 110 km); Figures 19c and 19d). The same is the case for the Helsinki area (Figure 19f). For Oscarshamn it is more difficult to draw any conclusion, but the flexural rigidity is definitely above 10^{24} N m (Figure 19e).

From tilts of paleoshorelines it is therefore suggested that the maximum ice thickness of Fennoscandia was between 2500 m (ice model 2) and 3750 m (ice model 4). The flexural rigidity in central parts of Fennoscandia is between 10^{23} and 2.5 x 10^{25} N m ($t_e \approx$ 20-110 km). The flexural rigidity at the western coast of Norway does not seem to exceed 10^{24} N m ($t_e \approx$ 50 km). If the glacier ice was considerably thicker than model 2, there seems to be a significant increase of the flexural rigidity toward the east.

Conclusions

The response of ice-load redistribution on Earth models with various lithosphere thicknesses is calculated to determine the elastic lithosphere rheology. For a relatively thin ice (2500 m in central areas) the resulting flexural rigidity was more or less uniform over Fennoscandia, 10^{23} N m (elastic thickness $t_e \approx$ 20 km). The seismic lithosphere has a significant thickness increase from the western coast of Norway toward the ice depocentre [*Panza*, 1985], from 110 km at the coast to approximately 200 km below the Baltic Sea. It is thus reasonable to assume a similar lateral change in the flexural rigidity as well. A flexural rigidity of 10^{23} N m is therefore regarded as a minimum value for central Fennoscandia.

The tilts of the paleoshorelines sets an upper bound of the flexural rigidity, 2.5 x 10^{25} N m ($t_e \approx$ 110 km). The Fennoscandian uplift data also suggest a flexural rigidity of maximum 10^{24} N m ($t_e \approx$ 50 km) in western Norway. If the glacier ice was thicker than model 2, there seems to be a significant increase of the flexural rigidity from the western coast of Norway to the Baltic Sea. The most likely ice model gives a flexural rigidity of 10^{23} N m (elastic thickness $t_e \approx$ 20 km) at the Norwegian coast, increasing to above 10^{24} N m ($t_e \approx$ 50 km) in central parts of Fennoscandia.

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